

EFFECTIF ASPECTS OF COMPLEX HYPERBOLIC VARIETIES

Aber Wrac'h (Brittany, France), September
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List of scientific talks and abstracts

M.BRUNELLA: Entire curves in real hypersurfaces

Abstract: Let X be a surface of general type and let $f : C \rightarrow X$ be an entire curve. We show that if $f(C)$ is contained in a real analytic hypersurface then $f(C)$ is actually contained in a complex algebraic curve.

(J'insiste sur le fait que ce resultat n'a presque rien a voir avec l'hyperbolicite (si ce n'est que pour l'usage de certains resultats bien connus). En fait, c'est un resultat sur les singularites des hypersurfaces Levi-plates analytiques reelles, de nature completement locale.)

R.de JONG: On an isomorphism of line bundles connected with Arakelov's proof of rigidity

Abstract: For his proof of rigidity for curves over function fields with given locus of bad reduction, Arakelov uses Weierstrass points to achieve ampleness of the relative dualizing sheaf. Combining his isomorphism involving Wronskian differentials with an isomorphism observed by Mumford one arrives at a certain explicit ample line bundle on the moduli space of stable curves. We study some of the properties of this line bundle.

J.-P. DEMAILLY: On the algebraic structure of the ring of jet differential operators

Abstract: New results on the algebraic structure of jet differential operators on surfaces will be presented : calculation of generators, relative base locus, Chern classes. The ultimate goal is to calculate the asymptotics of

the number of sections on an arbitrary subvariety of the jet spaces, leading potentially to hyperbolicity results of a very general nature.

J.DIVERIO: Existence of invariant jet differentials via holomorphic Morse inequalities

Abstract: Let $n = 2, 3, 4, 5$ and let X be a smooth complex projective manifold of dimension n and degree d . We determine an effective lower bound for d in order to have existence of global invariant jet differentials on X . This is done by using the algebraic version of holomorphic Morse inequalities on a particular subbundle of the bundle of invariant jet differentials.

J.DUVAL: On Brody's Lemma

T.ECKL: Projective complex manifolds with non-positive holomorphic bisectional curvature

Abstract: We present an approach of Wu and Zheng to prove a structure theorem for complex manifolds with non-positive holomorphic bisectional curvature. In particular we point out the essential role of the maximal foliation tangent to the null directions of the curvature on such manifolds and suggest an algebraic method to construct such foliations using the Harder-Narasimhan filtration of the tangent bundle.

J.ElGoul: Almost ampleness of the cotangent bundle of some surfaces of general type

A.KODAMA: An intrinsic characterization of the unit polydisk

Abstract: We give an intrinsic characterization of the unit polydisk in the complex euclidean n -space from the viewpoint of the holomorphic automorphism groups.

J.J.LOEB: Renormalization of harmonic functions and hyperbolic tube domains

Abstract: The Zalcman renormalization method is used to prove renormalization of non normal sequences of harmonic functions (or maps) to non-constant affine functions. We give some applications to hyperbolicity criteria for tube domains. (Reference: The paper: Applications harmoniques et hyperbolicite de domaines tubes, accepted in Enseignement Mathematique. A preliminary version is on arxiv).

S.LU: Some Chern number inequalities and their applications to varieties of general type

Abstract: We will review and strengthen some inequalities of Miyaoka-Yau type in order to obtain effective bounds on codimension-one images of Calabi-Yau varieties in varieties of general type.

F.MANGOLTE: Surfaces de Del Pezzo singulières réelles et variétés de dimension 3 fibrées en courbes rationnelles (Travail en collaboration avec Fabrizio Catanese)

Résumé : Soit $W \rightarrow X$ une variété projective non singulière réelle de dimension 3 fibrée en courbes rationnelles. On suppose que $W(R)$ est orientable. Soit N une composante connexe de $W(R)$. D'après Kollar, N est alors essentiellement une variété de Seifert ou une somme connexe d'espaces lenticulaires. Soit k un entier défini de la façon suivante : Si $g : N \rightarrow F$ est une fibration de Seifert, on note k le nombre de fibres multiples de g . Si N est une somme connexe d'espaces lenticulaires, on note k le nombre d'espaces lenticulaires différents de $P^3(R)$.

Théorème 1: Si X est une surface géométriquement rationnelle, alors k est majoré par 4.

Théorème 2: Si de plus F est difféomorphe au tore $S^1 \times S^1$, alors $k = 0$.

Ces résultats répondent par l'affirmative à deux questions de Kollar qui avait montré en 1999 que k était majoré par 6. On déduit ce théorème d'une analyse fine de certaines surfaces de Del Pezzo singulières avec singularités de Du Val.

T.MATSUSHIMA: Radial cluster set of bounded holomorphic maps in the unit ball of C^n

Abstract: We will construct a bounded holomorphic map whose radial cluster set at every point in arbitrarily given sequence in the boundary is a direct product of balls.

J.MERKER: Fifth order Demailly-Semple jets in dimension two

Resumé: Les resultats comprennent : etude des jets d'ordre 4 et 5 en dimension 2 ainsi que des jets d'ordre 4 en dimension 3 ; etude d'un sous-fibre dont la caracteristique d'Euler croit plus rapidement que celle du fibre de Demailly-Semple.

C.MOUROUGANE: Hodge metrics and the curvature of higher direct images

Abstract: Using the harmonic theory developed by Takegoshi for representation of relative cohomology, and the framework of computation of curvature of direct images bundles by Berndtsson, we prove that the higher direct images by a smooth morphism of the relative canonical bundle twisted by a semi-positive vector bundle are locally free and semi-positively curved, when endowed with a suitable Hodge type metric.

J.NOBUCHI: A survey of estimates of SMT-type for holomorphic curves

Abstract: An estimate of Nevanlinna's SMT-type which is once established is very effective for the study of degeneracy of holomorphic curves and the Kobayashi hyperbolicity problem. Beginning with Cartan's SMT, I will survey the estimates of SMT-type for holomorphic curves so far obtained, referring related results. I will mention open problems in this direction, too.

E.ROUSSEAU: On the Kobayashi Conjecture

Abstract: I will describe some results obtained around the Kobayashi conjecture which predicts the hyperbolicity of generic hypersurfaces of large degree in the projective space and the hyperbolicity of their complements.

M.RU: A fundamental inequality for holomorphic curves into projective varieties

Abstract: In the talk, I'll present the following theorem:

Main Theorem: Let $X \subset \mathbb{P}^N(\mathbb{C})$ be a smooth complex projective variety of dimension $n \geq 1$ and degree d . Let $f : \mathbb{C} \rightarrow X$ be an algebraically non-degenerate holomorphic map, and let $\mathbf{f} = (f_0, \dots, f_N)$ be the reduced representation of f . Define, for every $z \in \mathbb{C}$,

$$c_j(z) = \log \frac{\|f(z)\|}{|f_j(z)|}, 0 \leq j \leq N,$$

and let $\mathbf{c}(z) = (c_0(z), \dots, c_N(z))$. Denote by $e_X(\mathbf{c})$ the Chow weight of X with respect to \mathbf{c} . Let L be an ample line bundle and let $c_1(L)$ be the Chern form of L . Then, for every $\epsilon > 0$,

$$\frac{1}{d(n+1)} \int_0^{2\pi} e_X(\mathbf{c}(re^{i\theta})) \frac{d\theta}{2\pi} \leq (1 + \epsilon) \int_{r_0}^r \frac{dt}{t} \int_{|z| < t} f^* c_1(L),$$

where the inequality holds for all $r \in (0, +\infty)$ except for a possible set E with finite Lebesgue measure.

Various consequences of the theorem, including the recent solution to the Shiffman conjecture by the author, will also be discussed.

N.SIBONY: On global properties of holomorphic foliations in CP^2

D.D.THAI: Geometry of domains in C^n with noncompact automorphism groups

Abstract: In this talk, we would like to present two problems:

1) Characterization of domains in C^n by their noncompact automorphism groups.

2) Concerning the Green-Krantz-Conjecture, we study the parabolicity of domains in C^2 with boundary points of infinite type by showing that, generally, there is no parabolic orbit accumulation point of infinite type for these domains.

T.V.TRAN: Value Distribution Theory and its applications to the uniqueness problem

Abstract: This talk contains two parts. In the first part, we discuss the Second Main Theorem for algebraically nondegenerate meromorphic mappings into CP^n with moving hypersurfaces and multiplicities are truncated. This part is a joint work with Gerd Dethloff. In the second part, unicity theorems of meromorphic mappings with few hyperplanes are given. This is a joint work with Si Duc Quang.

H.TSUJI: Canonical singular hermitian metrics on relative canonical bundles

Abstract: I would like to explain how to construct a singular hermitian metric \hat{h}_{can} on $k_X + \Delta$ for a KLT pair (X, Δ) such that

- (1) \hat{h}_{can} is uniquely determined by (X, Δ) .
- (2) \hat{h}_{can} is an AZD of $K_X + \Delta$.
- (3) The curvature current of \hat{h}_{can} is semipositive on a projective family of KLT pairs.

I would like to discuss application of the metrics such as

- (1) Deformation invariance of (log) plurigenera.
- (2) Semipositivity of multirelative canonical bundles which relates to the Iitaka conjecture.
- (3) Moduli problem for polarized varieties.

J.WINKELMANN : On Brody Curves

M.ZAIDENBERG: Hyperbolicity of general deformations

Abstract: The Kobayashi problem suggests that generic hypersurfaces of degree $2n - 1$ in P^n are Kobayashi hyperbolic. So far the optimal bound in this problem has not been reached. Therefore, constructing small degree examples is of interest. We will introduce a deformation method for constructing small degree examples. This is joint work of the speaker with B.Shiffman.