Contemporary Mathematics
Volume 00, 0000

Correction to "A Real Algebraic Vector Bundle is Strongly Algebraic whenever its Total Space is Affine"

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M. Coste pointed out to me that the proof of the Lemma of [**2**] is incomplete. Indeed, with notation as in [**2**], the natural map from an $\mathcal{R}(X)$-module M into the $\mathcal{R}(X)$-module of global sections of the sheaf $\widetilde{M}$ is not necessarily surjective. To convince oneself, one takes $X = R^2$ and $M = \mathcal{R}(X)/(x^2(x-1)^2 + y^2)$. Let $U = X \backslash \{(0,0)\}$ and $V = X \backslash \{(1,0)\}$. Consider the exact sequence associated to the covering $\{U, V\}$ of X:

$$0 \to \Gamma(X, \widetilde{M}) \to \Gamma(U, \widetilde{M}) \times \Gamma(V, \widetilde{M}) \to \Gamma(U \cap V, \widetilde{M})$$

Since $x^2(x-1)^2 + y^2$ is invertible on $U \cap V$, the group $\Gamma(U \cap V, \widetilde{M})$ is 0. Hence, the map from $\Gamma(X, \widetilde{M})$ into $\Gamma(U, \widetilde{M}) \times \Gamma(V, \widetilde{M})$ is an isomorphism. Since the natural map from $M \times M$ into the product $\Gamma(U, \widetilde{M}) \times \Gamma(V, \widetilde{M})$ is injective, the natural map from M into $\Gamma(X, \widetilde{M})$ is not surjective.

The preceding observation shows that—to say the least—an argument is missing in the proof of the Lemma of [**2**]. Indeed, in that proof, I claimed implicitly that the global sections s_{ij} of $\widetilde{M}$ are elements of M.

Now, the Lemma is only invoked at the end of the proof of the Theorem of [**2**]; at the point where we know that $\Gamma(\cdot, \xi^\curlyvee)$ is isomorphic to $(I/I^2) \otimes_{\mathcal{R}(X)} \mathcal{R}_X$. This fact already implies that ξ is strongly algebraic. Indeed, for each maximal ideal $\mathfrak{m}$ of $\mathcal{R}(X)$ the $\mathcal{R}(X)_\mathfrak{m}$-module $(I/I^2)_\mathfrak{m}$ is free of rank n, where n is the rank of ξ. Moreover, I/I^2 is of finite type as $\mathcal{R}(X)$-module, $\mathcal{R}(E)$ being Noetherian. Hence ([**1**], Theorem II.5.2.2) the $\mathcal{R}(X)$-module I/I^2 is projective and of finite type. This implies that $\xi^\curlyvee$ is strongly algebraic. It follows that ξ is strongly algebraic. $\square$

1991 *Mathematics Subject Classification.* 14P05.

References

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